[illegible]THMOTECH[illegible]

OR THE

[illegible]

CALLED

LOGARITHMS

To Twenty five Places,

FROM A

GEOMETRICAL FIGURE,

With Speed, Ease and Certainty.

The like not hitherto Published.

By *EUCLID SPEIDELL*, Philomath.

LONDON,

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LOGARITHMOTECHNIA:

OR, THE

Making of Numbers

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By AUGUSTUS SPEIDLER, Polycarb.

LONDON,

Printed by Leon Lyar, in the Ayrshire, and are to be [illegible]

To the Right Honourable

Sir *NICOLAS BUTLER*, Kt.

One of His Majesty's Most Honourable Privy Council,

The Honourable

HENRY BROWNE, Esq;

The Honourable

Sir *DUDLEY NORTH*, Knt.

Sir *JOHN WERDEN*, Bar.

WILL. CULLIFORD, Esq;

COMMISSIONERS

Of His MAJESTY'S

CUSTOMS.

IN most Humble and Grateful Acknowledgment of your Honours Favours to me, in the Service of His Majesty's Customs, under your

Honours Management; this ensuing Treatise, being the Product of some leisure Hours from that Employment, I hold under your Honours, is with all Duty and Submission, humbly Dedicated, by

Your Honours

Most Faithful and

Obedient Servant,

EUCLID SPEIDELL.

Geometrical Logarithms.

CHAP. I.

ABout the Year 167½, being in Company with *Michael Dairy*, a Citizen of *London* (who had for most part of his life time addicted himself to Mathematical Studies, and hath published divers practical Pieces of several Parts of the Mathematicks, of good Use and Delight) and discoursing about making Hyperbolical Logarithms, I desired him to give me a Rule to make the Hyperbolical Logarithm of 10, from the Consideration of an Hyperbola inscribed within a Right Angled Cone, who gave me this Rule following.

To the Number proposed, *viz.* 10, add an Unite, and substract from it an Unite, and there will be a result of $\frac{9}{11}$: Then divide 1, or 100000000, *&c.* by $\frac{9}{11}$, which is 818181818, *&c.* which Cube *in infinitum*, and divide every one of them (which will be a Rank of Proportional Numbers) by the proper Indices of their respective Powers; that is to say by 3, 5, 7, 9, 11, *&c.* Then the Addition of all those Quotes will make the Logarithm of 10.

Finding then that 10 divided by $\frac{9}{11}$, maketh 818181818181818181, *&c.* and to Cube it in *infinitum*, was very difficult, I rejected the Rule, and thought it then not much more easie than *Brigg*'s way: Neither did he tell any Reason or Demonstration for the said Rule; and because in this Example, I found it so intricate, I did not much care to prosecute it, but neglected it. Not long after he departed this Life; and since his Death resuming the said thing, and trying if it were serviceable in any other part of the Hyperbola, I soon found it a Jewel, and could make the Hyperbolical Logarithm of 10 at twice, that is to say, from two parts numbered in an Asymptote, whose Fact is 10, with ease, certainty and delight, and have made the Hyperbolical

Logarithm of 2 to 25 places, in order to see if the learned and laborious *Henry Brigg*'s Logarithms were true to 15 places, which were made after a most laborious and difficult way of Extracting Square Roots, and, as I have heard, was the work of eight Persons a whole Year, and that without any proof, but only if any two or more agreed in their Extractions, Line by Line, Step by Step, it was taken *de bene esse*, which was a work of very great pains and uncertainty: However, they did effect it, and I do find they made the Logarithm of 2 to 15 places very true, as by my Operation, hereafter following, will appear, being done to 25 Places, and afterwards from these Hyperbolical Logarithm deduced *Brigg*'s Logarithms; both which Figurative Operations were performed and examined by me in 8 Hours time. I took this pains to make the Hyperbolical Logarithm to 25 Places; in order also, to see if the most ingenious and laborious *James Gregory*'s Hyperbolical Logarithm would agree with this of mine, which he hath in his *Quadratura Circuli & Hyperbola*, Printed at *Padua*; but I find that his Logarithm of 2 corresponds with mine but to 17 Places, I must confess, I did not take the Pains to raise the Logarithm of 2 to 25 Places, according to the Doctrine he hath delivered in that most Learned Piece, but am contented that this easie and certain way I deliver here, and by the Operation thereof the Hyperbolical Logarithm of 2 to 25 Places, is as true in the last as in any where, and may be examined in a few Hours; so that any Body, if he please, may be his own Examiner and Judge, if this Way be not easie, certain and speedy.

Having made several Logarithms for Digit Numbers, and Mixt Numbers, as for $1\frac{1}{2}$, $1\frac{1}{3}$, which are hereafter inserted. The Rule delivered by *Michael Dairy*, is of admirable use and benefit in Squaring the Hyperbola, and making Logarithms from it.

Some time since the death of the said *Michael Dairy*, I shewed unto Mr. *John Collins*, whom I knew had been a great Familiar and Friend to the said *Michael Dairy*, the Figurative Work of my making, the Hyperbolical Logarithms, according to the said *Dairy*'s Rule, who seemed very well pleased with it, acknowledging it to be the speediest way could possibly be, of Squaring the Hyperbola, and making the Logarithms from it, and after a little pausing upon it, replyed, That *Dairy* must have

have this Rule out of the said *James Gregory*'s Works. I made answer, Not from his said *Quadratura Circuli & Hyperbolæ*: He answered, No, from his *Exercitationes Geometricæ*, printed at *London*, 1668, a Book I had not seen nor heard of till then: And as the said Mr. *Collins* had been always very frank and free to communicate any Mathematical thing to me, so I held my self obliged to acquaint him first with this Work. He seemed to admire, That *Michael Dairy* should keep such a thing from him, who had been so great a Familiar with him in these Studies. Not long after my discovery hereof to the said Mr. *Collins*, he also departed this Life; whose Death, all that were Mathematical, and knew him, lamented not a little: For he was not only excellent in Mathematical Arts and Sciences, but of a very good, affable and frank Nature to Communicate any thing he knew to any Lover and Enquirer of those things, and hath left behind him those Mathematical Works which will continue his Fame amongst the Lovers and Students therein. He also in his Life-time, promoted the Publishing of other Men's Mathematical Works; as the Elaborate *Algebra* of the Learned *John Kersy*, who was my Father's Disciple about 164[illegible]: And also of the Learned *Baker*'s Algebra, and several others. He was a Man of great correspondence with Mathematical Persons in foreign Parts, and thereby could give Information of any New or Old Mathematical Book; and till my Acquaintance with him, I was ignorant of Foreign Authors; being but young when my Father dyed, and not then having taken any Pains in these Studies: So that by the said *Collins*'s Information and Means, I have heard of, and seen, some Foreign Mathematical Authors of Note and Esteem.

After the said Mr. *Collins* had told me of *James Gregory*'s said *Exercitationes Geometricæ*, sold by *Moses Pitts* in *Paul*'s Church-Yard, I bought there one of them; and do find that *Michael Dairy* had deduced this Rule from the said Book: Wherein the said *James Gregory* hath made the Squaring of the Hyperbola, an Exercise Geometrically demonstrating the Quadrature of the Hyperbola, some time before published by the Industrious and Lucky *Nicholas Mercator*; who by the happy discovery of some Properties in the Hyperbola, hath made all the Ways of Squaring the Hyperbola flowing from the same, very easie, certain and delightful: And because neither of them have exemplified

their Doctrine and Rules with Figurative Work, ſo large as to 25 Places. I have here, to Illuſtrate their Admirable Works, inſerted divers Figurative Operations, whereby the Reader and Student may ſee, and have that Satisfaction in Fact and Operation, which is ſo pleaſing and deſirable by every one.

I ſhall not here trouble the Reader with any Sections of the Cone, whereby he may ſee the riſe and geniture of an Hyperbola, from that Body, but content my ſelf to ſhew him from a Square and an infinite company of Oblongs on a Superficies, each Equal to that Square, how a Curve is begotten which ſhall have the ſame properties and affections of an Hyperbola inſcribed within a Right Angled Cone: And ſeeing a Curve made after this manner following, doth become ſuch an Hyperbola, the Doctrines and Analogies delivered and diſcovered by thoſe two Ingenious Artiſts *Mercator* and *Gregory*, may be applyed to this Curve ſo often as need and occaſion doth require.

And not to detain the Reader any longer from knowing how to make this Curve, we proceed to deſcribe the ſame accordingly.

There is a Square ABCD, whoſe Side or Root is 10, let DB be prolonged in *infinitum*, and continually divided equally by the Root, or DB, and thoſe Equal Diviſions numbered by 10, 20, 30, 40, 50, 60, 70, *&c.* in *infinitum*: Upon theſe Numbers let Perpendiculars be erected, which call Ordinates, and each of thoſe Perpendiculars of that length, that Perpendiculars let fall from the aforeſaid Perpendiculars to the Side or Baſe CD (which call Complement Ordinates) the Oblongs made of the Ordinate Perpendiculars, and Complement Ordinate Perpendiculars may be ever Equal to the Square AD, which is eaſily done thus, for it is $\frac{100}{20}$ $\frac{100}{10}$ $\frac{100}{40}$ $\frac{100}{10}$ *&c.* produces the Length of the Ordinate Perpendiculars; for 100 divided by 20 maketh 5 for the Length of the Ordinate Perpendicular 20 E. And 100 divided by 30, giveth 3333333, *&c.* for the Ordinate Perpendicular 30 F. And 100 divided by 40 produceth 25 for the Ordinate 40 G, and ſo of the reſt. And Geometrically it is as 20 D is to BD, ſo is AB to AH, equal to 20 E, as before, for that the Angle ACH is equal to C 20 D, and ſo of the reſt. And for the Length of the next Ordinate, you ſay, as 30 D to BD; ſo AB to AK, which is alſo Equal to 30 F. And for the Ordinate 40 G, ſay, as 40 G to BD, ſo AB to AM, which will be

Equal

equal to 40 G, and ſo of all the reſt, whereby you have all the Perpendiculars upon the prolonged Side DB, both Geometrically and Arithmetically; the ſame Proportion is to be obſerved for any Intermediate Parts.

Now, for all the Perpendiculars which are let fall from the aforeſaid Perpendiculars or Ordinates to the Baſe CD, which call Complement Ordinates, the Geometrical Proportion for NE, equal D 20 is as HA to AC, ſo CD to CL 20 equal to NE, and for the Complement Ordinate OF equal D 30, it is as KA to AC, ſo CD to D 30 equal OF, and ſo of the reſt. Now for NE Arithmetically, ſay, as 5 to 10, ſo 10 to 20 equal to NE, equal to D 20; and for OF, ſay, as 33333333 to 10, ſo 10 to 30 equal OF, which is equal to CL 30, and for PG equal to D 40, ſay, as 25 is to 10, ſo 10 to 40 equal to PG, equal to D 40; and ſo for all the reſt of the Complement Ordinates ſtanding upon the Baſe CD, whereby it doth appear, That all the Oblongs made of the Ordinates, and Complement Ordinates are each of them equal to the Square AD, which is here 100; for the Oblong ED being made of E 20 and D 20, is by the 13 of of the 6 Euclide equal to the Square AD, for Q 20 is a Mean Proportional between D 20, and 20 R, and Q 20 is found to be equal AB, ſo is the Oblong or Parallelogram ED equal to the Square AD, and the like Demonſtration ſerves for all the Oblongs or Parallelograms ſtanding upon the Baſe CD, by the Tips or Angular Points of thoſe Parallelograms, or from the Ends of all the Ordinates ſtanding upon 20, 30, 40, 50, 60, 70, *in infinitum*; draw the Curve Line from A towards E, ſo ſhall you deſcribe the Curve AEFGS, which Curve you ſee is begotten without any conſideration or reſpect to the Section of a Cone, and yet becomes the ſame in all reſpects, to have the ſame Affections and Properties of an Hyperbola derived from the interſecting of a Right Angled Cone, as ſhall be ſhewed in the next Chapter.

You may obſerve the Complement Ordinate NE, being equal to D 20, is equal to twice Radius. And if CD be made the Radius of a Circle, then is NE equal to D 20, equal to the Tangent of twice Radius, for D 20 becometh the Tangent of twice Radius. Alſo it is manifeſt that the Complement of the Tangent equal to twice Radius, is alſo equal to half the Radius; That is, the Tangent Complement of D 20, is 20 E equal to 5.

And

And seeing the Radius is ever a mean Proportion between the Tangent and the Tangent Complement, therefore each Oblong is equal to the Square AD.

CHAP. II.

IN the former Chapter, we have shewed the begetting of a Curve, without any regard to the Section of any Solid Body; and now it remaineth to prove that this Curve hath the same Properties and Affections that an Hyperbola, deduced from the Section of a Right Angled Cone.

I remember some time before the death of *John Collins*, he told me, It was a great Work of the Learned *Vincent* or *Magnan*, to prove that Distances reckoned in the Asymptote of an Hyperbola, in a Geometrical Progression, and the Spaces that the Perpendiculars, thereon erected, made in the Hyperbola, were equal the one to the other. This Property is now very well known, the Hyperbola hath, and this Curve hath the same Property; which is discernable almost *intuitu*. In the Hyperbola, they call the Prolonged Line DB *in infinitum*, from the Point B, an Asymptote. And here in this prolonged Line from B, on 20, 40, 80, 160, 320, 640, *&c.* let the Ordinates touch the Curve in EFGS, *&c.* I say, That those Trapezias with the Curve Line (or Hyperbolical Spaces) are all equal the one to the other. In the Right Lined Trapezias thereon, it is manifest they are all equal the one to the other by several Propositions of the 6th Book of *Euclid*: For in the Right Lined Trapezia ZEAB the side AB is twice EZ, and by the former Chapter it was found that GY is half EZ, by saying, As AB to EZ, so EZ to GY. And the Right Lined Trapezia ZEAB shall be therefore equal to 75: Now, forasmuch, as in the Right Lined Trapezia YGEZ the Base of that YZ, is double to ZB, but the Perpendiculars are in the *Ratio* of AB to EZ; for as before, it is as AB: EZ:: EZ: GY, therefore the Right Lined Trapezia YGEZ equal to the Right Lined Trapezia ZEAB, and so will all Right Lined Trapezia's, so Based and Perpendicular'd, be equal the one to the other. The Trapezia LYEZ is equal

to

to the Square AD, because ZV · ZE is equal to AB · AB, as in the foregoing Chapter, the Oblong GZ is half the Parallelogram LZ, and the Triangle GEI half the Parallelogram LI. Now the Parallelogram GZ + the Triangle GEI (half the Parallelogram LI) is equal to the Right Lined Trapezia ZEAB, for in Numbers 20 · 25 = 50 + half 50 equal to 75.

Thus you see the Right lined Trapezias numbred upon the ~~prolonged side in a Geometrical Proportion are~~ equal the one to the other; it remaineth now to prove the mixed Trapezias, that is, the Trapezias standing upon the same basis but joyned aloft with this Curve are also equal the one to the other.

First let it be observed, That these Curvilined Trapezias (or Hyperbolical Spaces) are ever less than the Right lined Trapezias, because all the Points in the Curvilined Trapezias fall within the Right line that joyns the Right lined Trapezias: And is thus proved in the Right Lined Trapezia BZEA; let there be in the Base Z B upon the point 5 Erected a Perpendicular to touch E A in T then is T 5 equal to A B less a T which is half QE, that is, 10 less 2, 5 equal 7, 5 = T S. But by the foregoing Chapter, if a Perpendicular be erected upon the said Point 5 to V, (to touch the Curve in V) so that the Parallelogram VD shall be equal to AD, as in the former Chapter, then will it be AD divided by D5 = 5 V, which is 100 divided by 15, produceth 6.666666 for the true Length of 5 V, whereas before 5 T is 7, 5. By the same means may all the intermediate Points in this Curve Line EVA be found to fall within the Right Line AE, that is, between the Line EA and ZB, and therefore the Right Lined Trapezia ZETAB greater than the Curvilined Trapezia (or Hyperbolical Space) ZEVAB.

Now, forasmuch as we have proved that the aforesaid Right Lined Trapezias are ever equal the one to the other, it will now follow, That seeing the Curve passing by all those Points which are Extremities of the Right Lined Trapezia, (as well as the Curvilined Spaces, being upon the same Bases always) and this Curve being generated continually by one and the same Ratio, as in the former Chapter. That therefore the Curvilined Trapezias standing upon Geometrical Proportional Bases, shall be also equal the one to the other, which is the Affection and Property of the Hyperbola. And so the Doctrines and

Precepts

Precepts delivered by those two Famous Geometers, *Mercator* and *Gregory*, for the Squaring of the Hyperbola, be applyed to this Geometrical Curvilined Figure; and from it derived Logarithms, which may be called Hyperbolical Logarithms.

The way and means to find the Hyperbolical Spaces in Numbers, shall be shewed in the following Chapters.

CHAP. III.

IN this Chapter we will consider that most admirable discovery (I suppose *Mercator* made) upon drawing the Diagonal CB, which by construction cutteth all the Perpendiculars standing upon the Base CD at Equal Angles, and in such Distances from the Base CD, as doth unravel the Mystery of his infinite Series, and make the Quadrature of the Hyperbola more easie and certain than any I ever saw or heard of.

The Diagonal CB being drawn doth give the first Term of a Geometrical Progression or Infinite Series between 10 and 20, or 30, 40, 50, 60, 70, 80, 90, *&c.*

That is to say, Would you know the first Term of an Infinite Series (or Numbers Geometrical Proportional continued) between 10 and 20, the summ of all which shall be just 20. Having from *Z* drawn the Line ZC, to cut BA in H, which taken off, and applyed to CD, from C to N equal NB, because the Angle BCN is equal to the Angle CBN: I say that NB is the first Term of an Infinite Series between CD equal AC, and the Perpendicular NE equal DZ, which may be done by Squaring AC, and dividing it by the Side or Number given, the Complement whereof to 10 is the first Mean or Term of that Infinite Series, so shall the first Term of the Infinite Series between 10 and 20 be found 5: Thus in Numbers, $10 \times 10 = 100 \ldots = 5$, the Complement whereof to 10 is 5, equal CN, equal NB for the first Term of an Infinite Series between 10 and 20, whose summ

is 20

is 20, as by the Arithmetical work in the Margent where *a, b, c, d, e, f, &c.* are a Rank of Geometrical Progressional Numbers, whose infinite Sum would make but 20, and is demonstrated by the

And in Numbers thus. As 10 less 5 is to 10 what 10? the Quotient will be found 20 for the whole Summ of that infinite Series between 10 and 20 whose first Term is 5

In like manner would you know the first Term of an infinite Series between 10 and 30 Divide the Square of ac = 100 by 30 the Quotient will be found 3333333 whose Complement to 10 is 6666666 I say, That 6666666 is the first Term of an Infinite Series between 10 and 30, as by the Arithmetical Operation in the Margent; And briefly thus, as 10::3333333=6666666: 10:: 10: 30 so is 30 the whole Summ of all those infinite Progressional Numbers between 10 and 30. In the figure you draw the line ϕC which cutteth BA in K, I say that BK transferd from C to O equal Or is the first Term of an infinite Series between AC and OF equal Dϕ. And PS be the first Term equal 7, 5 equal PC between AC and PG equal DY = 40 between 10 and 40, for as before 10 —7, 5 =2, 5: 10:: 10 = 40 so is 40 the whole Summ of an infinite Series between 10 and 40, whose first Term is 7, 5 and so of all the rest. On this great Mystery depends much the following squaring of the Hyperbola and hath made it so intelligible and easie.

a.	10
b.	5
c.	2. 5
d.	1. 25
e.	. . . 625
f.	. . . 3125
b.	. . . 15625
i.	 78125
k.	 390625
l.	 1953125
m.	 9765625
	19. 99. 0234375
a.	10.
b.	. 6. 666666666
c.	. 4. 444444444
d.	. 2. 962962962
e.	. 1. 975308641
f.	. 1. 316872427
g.	. . . 877914951
b.	. . . 585276634
i.	. . . 390184422

Hence you may Note, That if you would know the first mean of an infinite Series between any other Number, and that Number doubled, tripled, quadrupled, *&c.* you may from this Root 10 deduce it, as for Example, let CD be 12 and I would know the first Term of an infinite Series between 12 and 24 the double of 12 I say as 10 is to 5 what 12 *facit* 6. for supposing CD = 10 it was before found 5. Therefore between 12

and 24 you say 10 : 5 :: 12 : = 6 for the first mean of an Infinite Geometrical Progression between 12 and 24. And is thus by the former Analogy proved. by saying as 12—6 = 6. : 12 : : 12 : = 24 so is 24 found to be the total Summ of an infinite Series between 12 and 24 as by the Operation in the Margent will appear.

a.	12
b.	6
c.	3
d.	1, 5
e.	. . 75
f.	. . 375
g.	. . . 1875
h.	. . . 9375
i.	. . . 46875
k.	. . . 234375
l.	. . . 1171875
m.	 5859375
	23, 994140625

Also if it were required to find the first mean between 12 and three times that number *viz.* 36. Say as 10: 6666666: 12:: 8. And so by Tabulating or working this Example as you do the former the total Summ of that infinite Series between 12 and 36 (the first Term being found 8) will amount to 36 : And for proof you say 12 — 8 = 4: 12 : : 12 : = 36 so is 36 the whole Summ of that infinite Series or Geometrical Progression between 12 and 36, the first Term being 8 as was desired.

If it shall be required to know an infinite Series between 10 and any other Number, as to know the first Term between 10 and 15 that is, between DC and D5, draw 5C, and it cutteth BA in E, I say that BE is the first Term of an infinite Series between 10 and 15 as by the Operation in the Margent, and as before taught 10 — = 6666666, which substract from 10 leaveth 3333333 for the first Term. And by the former Rule is proved thus; As 10—3333333 = 6666666 : 10 : : 10 : = 15. Thus may you find the first of any Term of an infinite Series between 10 and any other Number.

a.	10
b.	3333333333
c.	1111111111
d.	370370370
e.	123456793
f.	. 41152264
g.	. 13717421
h.	. . . 4572473
i.	. . . 1524157
k.	 508052
l.	 169350
m.	 56450
	14, 999971774

And if it should be desired to know the first Term of an infinite Series between any two other Numbers. As for Example, I would know the first Term of an infinite Series between 12 and 16, to do it Geometrically you must suppose BA = 12. And then counting 16 from D in the line DB prolonged by that Point, and C draw a line which will cut the line

line BA (now representing 12) in a point, which taken from *B*, will be the length of that line Geometrically, and Arithmetically it will be found by saying, as 16 D to DC, so 16 B = 4 to B 3 in the line BA numbred from B to A when BA is 12. In Numbers the Proportion stands thus, 16 : 12 :: 4 : = 3 which 3 is the first Term of an infinite Series between 12 and 16 as was required, and is manifest by the Operation in the Margent, which to prove by the foregoing Rules you say, as 12—3=9 : 12 :: 12 : =16; which is to say, as 9 to 12, what 12 *facit* 16 for the whole Summ of that infinite Series between 12 and 16, the first Term being found as before to be 3.

a.	12
b.	3
c.	...75
d.	..1875
e.	...46875
f.	...1171875
g.	29296875
h.	732421875
i.	....183105468 75
	15, 999938964843 75

Thus have you that great Mystery unfolded of finding Geometrically and Arithmetrically the first Term of a Geometical Progression with the whole Summ of that infinite Progression or Series between any two Numbers, which is the main thing I conceive that famous *Mercator* was so lucky in discovery thereof, and doth unravel the Mystery of squaring the *Hyperbola*, as will be manifest in the next Chapter following.

CHAP. IV.

I Observe from the said Learned *Gregory's Exercitationes Geometrica*, he giveth three Quantities or Spaces contiguous to the *Vertex* A, which shall be all equal theone to the other which is very true and perspicuous, and then shews how to find the *Areas* of them severally as in page 9, 10, 11, and 12 of said Book.

And here we must consider them all three before we come to understand *Dary's* Rule, which is but a Deduction from these, as will appear hereafter. And now I begin to consider the said three several Quantities or Spaces all contiguous to

the *Vertex* A, and of a different form, and yet equal the one to the other.

Let it therefore now be shewn those three several Spaces differing in form, and yet equal the one to the other contiguous to the *Vertex* A, which shall represent the Curvilined *Trapezia*, or Hyperbolical Space for 2. The first Curvilined *Trapezia* or Hyperbolical Space for 2 let be ZBAVE, which is intelligible *intuitu*. The second let be AVENC, which is equal to the former ZBAVE by the 43 of the 1 of *Euclid*, because the Parallelogram EZBH (BH being equal to ZE) is equal to the Parallelogram HACN (HA being equal to HB.) Now for as much as the Curvilined Triangle AVEH is common to both the said Curvilined *Trapezias* or Hyperbolical Spaces, it remaineth therefore, that these two Curvilined *Trapezias* or Hyperbolical Spaces ZEVAB, and AVENC are equal the one to the other. And now to find out the third Curvilined Space contiguous to the *Vertex* A, and yet equal to either of the other two, but differing in form, doth require a little further consideration which from him is directed thus. And is manifest by the figure, divide BZ in two equal parts in 5. Then as before taught will it be as 5D : DC :: 5B : BK = DX make CX equal to Cπ, or to find Dπ it is as D5 : DZ :: DC : Dπ upon π erect a Perpendicular to touch the Return or Continuation of the Curve on the other side of A, from A towards Θ in Σ. So is this Curvilined Figure or Hyperbolical Space πΣAVX (differing in form) equal to either of the other two ZEVAB or CAVEN. And from finding the *Area* of this Curvilined Figure or Hyperbolical Space πΣAVX is derived *Dairy*'s Rule, which is but a Deduction from the finding of the *Areas* of the other two, as will hereafter appear.

Arithmetically DX is found by saying, as 15 to 10 what 10? *facit* 66666666, and 10 less 66666666 rest 33333333 for Cπ so is Xπ equal to 66666666, or Dπ may be found thus; as D5 : DZ :: DC : Dπ, which in Numbers is, As 15 : 20 :: 10 : = 1. 33333333 for Dπ.

James Gregory in the 4 Proposition page 10, 11 of his *Exercitationes Geometrica* doth contemplate first, the Second of these three Curvilined *Trapezia* or Hyperbolical Spaces, that is to say, the Curvilined *Trapezia* or Hyperbolical Space CAVEN, and in that 4 Proposition after and long learned Demonstration

monſtration doth prove the Space CAVEN to be equal to his ſuppoſed quantity *, and then refers you to *Cavelerius*'s method of Indiviſibles, a Book I have not yet ſeen. Which briefly I conceive may be thus eaſily demonſtrated.

It is manifeſt that all the Perpendiculars let fall from the Ordinates (ſtanding upon the prolonged ſide DB) to the Baſe CD, doth not only deſcribe the Curve, but would alſo fill the whole Hyperbolical Space, were Number, and the Curve Definitive. And thoſe Perpendiculars let fall from the Ordinates (ſtanding upon DB prolonged, numbred 20, 30, 40, 50, 60, *&c.*) doth divide the Baſe DC, in $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$, *&c.* And whereas the Diagonal CB, by croſſing all thoſe Perpendiculars doth give the firſt Term of an infinite Series between the Root or Side AC (= NH) and the length of each of thoſe Perpendiculars ſtanding upon the Baſe CD. Therefore to know the Hyperbolical Space CAVEN, divide the Parallelogram CH, making it the firſt Term of an infinite Series by the *Ratio* of NB = NC to NH *in infinitum*, and each of thoſe Quotes or Proportional Numbers by 1,2,3,4,5,6,7,8, *&c.* alſo *in infinitum*. The Quotes of all the laſt Diviſions added, will give you the *Area* of the Hyperbolical Space CAVEN, and ſo for any other Curvilined or Hyperbolical Space ſtanding upon the Baſe CD, as by the Calculation following.

And before I handle any of the other two Curvilined Spaces differing in form, and yet equal to this Hyperbolical Space CAVEN, we will exemplify this Demonſtration in Operation, and the figurative work thereof, ſhall be the work of the next Chapter.

CHAP. V.

LET it be required to calculate the *Area* of the Curvilined *Trapezia* or Hyperbolical Space CAVEN to 15 Places, and hereafter you will ſee it done to 25 Places according to *Dairy*'s Rule in Chapter VII. but with greater diſpatch.

The

Thus by the Calculation do you find the *Area* of the Curvilined *Trapezia* or Hyperbolical Space CAVEN to the 15 place to be 693147180559945 equal to the Curvilined *Trapezia* ZEVAB, and also equal to the Curvilined *Trapezia* or Hyperbolical Space ΠΣAVX, the Calculation of the *Areas* of any part of these two latter shall be shewn hereafter which will differ in Operation, yet bring out the same Number, and in Calculating the last, we shall use *Dairy*'s Directions.

It having been before shewn that the Hyperbolical Space ZEVAB equal to the Curvilined *Trapezia* or Hyperbolical Space CAVEN is equal to the Curvilined *Trapezia* or Hyperbolical Space YGFEZ, that therefore the said ZEVAB is a Space or Quantity to represent the Logarithm of 2. So then the aforesaid Number 693147180559945 is an Hyperbolical Logarithm of 2. And having the Logarithm of 2. you have also the Logarithm of all the Powers of 2.

And by this Calculation you have not only gotten the Logarithm of 2, but gained also the Logarithm of 3, for if you add all the Quotes marked with this Asterism (*) the Addition of them shall be the half Summ of the Hyperbolical Logarithm of 3. agreeable to the *4th.* Consectary of the *4th.* Proposition, and first inference on the *5th.* Proposition of said *James Gregorye's Exercitationes Geometrica*, from whence 'tis plain that *Michael Dairy* had his Rule, as will appear more manifest after we have contemplated the two other Curvilined *Trapezias* or Hyperbocal Spaces ZEVAB and ΠΣAVX.

The Addition of the Quotes marked with * make 5493061443340 55 which doubled is 10986122886 68110 for the Logarithm of 3, and now having gotten the Logarithm of 3, you have also the Logarithm of all the Powers of 3, and of all the Composites of 2 and 3.

Again if you shall from 50 substract 125, and to that add 4166666666666, and from that substract 15625 and so on throughout, you shall have the Logarithm of the difference between 2 and 3, or the Logarithm of 1 and $\frac{1}{2}$ or 1 and $\frac{2}{3}$ correspondent to the inference on the 5 Prop. of *James Gregory*, all which shall be fully exemplifyed hereafter.

The Calculation of the Logarithm of 2, according to the Method before going is the ground work of all the Calculations following, and I shall only give the Calculation of one more

Space

Space to represent the Logarithm of 3 after that Method, though we have, you see, gotten already the Logarithm thereof, but supposing we had not, and were to find that first according to the said Method.

From F you let fall a Perpendicular as FO. So is the Curvilined *Trapezia* or Hyperbolical Space AEFOC equal to the Curvilined *Trapezia* or Hyperbolical Space AEF⊕B for the Oblong AEOC as before shewn will be equal to the Parallelogram EF⊕B, and the Curvilined Triangle AEFE common to both Parallelograms. Therefore the Curvilined *Trapezia* AEFOC equal to the Curvilined *Trapezia* or Hyperbolical Space AEF⊕B, to calculate whose *Area* which will be the Logarithm of 3 you proceed as followeth, which Work is but partly done, to shew the way thereof, the Logarithm of three being hereafter done to 25 places; but with far greater dispatch than this Method will permit.

666666666666666	:	I	666666666666666
444444444444444	:	II	222222222222223
296296296296296	:	III	.98765432098765
197530864197530	:	IV	.49382716049382
131687242798352	:	V	.26337448559671
87791495198902	:	VI	.14631915866484
58527663465934	:	VII	..8361094780848
39018443310630	:	VIII	..4877305288829
26012294873752	:	IX	..2890254985973
17341529915834	:	X	..1734152991583
11561019943888	:	XI	..1051001813081
7707346629258	:	XII	...642278885771

I have but gone twelve steps in the Calculation of the Logarithm of 3 after said Method, which will if it were added but give the Logarithm of 3 to five places. I have left it unfinished for the Exercise of those who shall take delight herein, and finish it throughout to the intent of making the Logarithm of 3 to fifteen places according to this Method. By adding the Quotes of so much as is done the first five figures will be 10986 correspondent to the Logarithm of 3; this Method being somewhat slow, I shall not Calculate the Logarithms of any other Numbers according to it. And by these two Examples the Reader may see enough to calculate any other Curvilined *Trapezia*

pezia or Hyperbolical Space ſtanding onely in or upon the Baſe CD equal to any Hyperbolical Space: reckoned in the prolonged ſide DB.

And ſo we will contemplate in the next Chapter the Affections and Properties of the Hyperbolical Space or Curvilined *Trapezia* AΣΘYC equal to the Curvilined *Trapezia* or Hyperbolical Space ABZEV.

CHAP. VI.

BEfore we ſhew how to calculate any part of the Curvilined *Trapezia* or Hyperbolical Space AΣΘTC, equal to the Curvilined *Trapezia* or Hyperbolical Space AVEZB. We will inſert Tables to illuſtrate the 1, 2 and 3 Propoſitions of *James Gregory's Exercitationes Geometrica.*

The First Table of Illustration.

I

A	5
B	25
C	125
D	625
E	3125
F	15625
G	78125
H	390625
I	1953125
K	9765625
L	48828125
Z =	666666666

II

The Rule to find Z.

5--25=25: 5 :: 5: =10=Z.

III

Z = A C E G I L

The Rule to find Z is

5--325=325: 5 :: 5: =

66666666 Impares = Z

IV

Z = B D F H K M

The Proportion to find Z

25--625=1875: 25 ::25: =

33333333 Pares = Z

V

The Rule is A-B C-D E-F G-H I-K L-M

25-625=1875:25::

25: =33333333

Equalis Paribus

VI

+A +B +C +D +E +F +G +H +I +K +L +M +N

The Proportion is

5+25=75 : 5 :: 5 : =33333333

Equalis Excessui Imparium supra omnes Pares.

VII

A +B C +D E +F G +H I +K L +M N+

The Proportion is

5-125=375: 5 :: 5 : =66666666

Equalis Paribus

VIII

+A +B +C +D +E +F +G +H +I +K +L +M +N

+A +B +C +D +E +F +G +H +I +K +L +M +N

2A+ 2C+ 2E+ 2G+ 2I+

The

The Explanation of the foregoing Table.

This Table consisteth of Eight Columns, The first is a supposed literal Rank of Quantities continually proportional. The second is of Numbers correspondent to the first in a Ratio, as 2 is to 1. Or 5 to 2,5. What 2,5? And so supposed continued *in infinitum*, with the Rule how to find out the whole Summ of those Numbers so continually proportional.

The third Column sheweth how to find the whole Summ of the odd Quantities or Numbers.

The fourth teacheth how to know the whole Summ of the even Quantities or Numbers.

The fifth telleth how to find the whole Summ of the Difference of the Quantities of the first Column.

The sixth supposeth A--B-|-C--D-|-E, and teacheth how to find the Solution thereof.

The seventh supposeth A-|-B-C-|-D--E, and giveth a Rule to resolve the same.

The eigth and last supposeth the whole Rank, first, Affirmative, and the second evenly or alternately less: And giveth a Solution thereof.

A further Explanation of this Table will be when we come to calculate the Area of any part of the Curvilined Trapezia, AΣΘYC.

The Second Table.

I	II	III	IV
Complement Ordinates standing upon CD (more than Radius) or Perpendiculars let fall from	Ordinates or Perpendiculars standing upon DB prolonged.	Arithmetical Complements.	What Proportion the Second Colum hath to Radius.
10	10	∞	
20	5	5	
30	3,33333333	6,66666666	
40	2,5	7,5	
50	2	8,	
60	1,66666666	8,33333333	
70	1,42857143	8,57142857	
80	1,25	8,75	
90	1,11111111	8,88888888	
10	1.	9.	

This Table consisteth of four Columns, The first is equal Spaces numbred in the side DB prolonged, or the Tangents greater than Radius.

The

The ſecond ſheweth the length of the Perpendiculars ſtanding upon the ſide DB prolonged; which are Tangents leſs than Radius, and by the Tops paſs the Curve or Hyperbolical line.

The third Column is the Arithmetical Complements.

The fourth Column ſheweth what Proportion the ſecond Column hath to Radius.

The Rectangle or Parallelogram of the firſt and ſecond Column is equal always to the ſquare AD.

This Table is of uſe to find Points to deſcribe the Curve or Hyperbolical line, or to examine if the Curve paſs through ſuch points as the Table mentions.

The making of this Table hath been formerly ſhewn, when it was taught how to deſcribe the Curve.

We now come to ſhew how to make a Table to find the length of the Baſes of the Compound Curvilined Trapezias, or Hyperbolical Spaces.

We call that a Compound Curvilined Trapezia or Hyperbolical Space, when AC is in the Middle of that Baſe.

So AC ſtanding upon the Middle of ΠX hath Perpendiculars or Sides Π☰ and XV, ſo is the Curvilined Trapezia Π☰AVX, to be hereafter underſtood a Compound Curvilined Trapezia or Hyperbolical Space, and will be ſhewn as followeth to be equal to the aforeſaid Spaces CAVEN, and to AVEZB for the Logarithm Space of 2.

And the Compound Curvilined Trapezia △ΘAVEN will be equal to the Curvilined Trapezia AVFOC, and to AVF♦B for the Logarithm Space of 3.

The Compound Curvilined Trapezia or Hyperbolical Space Π☰AVX we may prove to be equal to AVEZB thus, by the 43 of the firſt of *Euclid* the Parallelogram CK is equal to K 5, and the Curvilined Triangle AVK common to both, ſo then is AVKXC equal to AV5B. And the Parallelogram Π ♐ equal to the Square VZ, and the Curvilined Triangle ☰ ♐A equal to the Curvilined Triangle V☰E, and ſo the Compound Curvilined Trapezia Π☰AVX equal to the Curvilined Trapezia AVEZB for the Logarithm Space of 2. For by the *4th* Table following, look what Proportion the Perpendiculars or Sides of the Compound Curvilined Trapezias have one to the other, the like Proportion have the Sides or Perpendiculars of the other two Curvilined Trapezias.

So in this Compound Curvilined Trapezia ΠΣ and XV the Sides or Perpendiculars are in a Proportion as 2 is to 1 descending, or as 1 is to 2 ascending; so likewise in the Curvilined Trapezia CAVEN (equal to the aforesaid Compound Curvilined Trapezia ΠΣAVX) the Side or Perpendicular NE is double to CA. And also in the Curvilined Trapezia AVEZB (equal as before to the Compound Curvilined Trapezia ΠΣAVX) the Side or Perpendicular BA twice ZE as before taught.

Thus by the Ratio of the 2 Tables following may you make a Compound Curvilined Trapezia equal to either of the other two Curvilined Trapezias or Hyperbolical Spaces, and the calculating the Area of the Compound Curvilined Trapezias will be found to be of far greater Dispatch than the former Method, by which we shall make use of *Dary*'s Rule, or rather the learned *James Gregory*'s from his first Inference on his 5 Proposition.

We come now to insert the third Table, which is a Table of Ratios to find the Length of the Bases of the Compound Curvilined Trapezias.

You may note, that in all the three different sorts of Curvilined Trapezias or Hyperbolical Spaces equal the one to the other, if on the Middle of their Bases, you shall erect Perpendiculars to touch the Curve the greater part or segments in each is equal to either greater segment of the other, and so is the lesser part or segment of the one equal to the lesser segment of either of the other.

The

The Third Table.

Being a Table of Ratios to find the Length of the Baſes of the Compound Curvilined Trapezias or Hyperbolical Spaces.

I	II	III	IV	V	VI
5D	DC	DC	DX		
15	10	10	,666666666	For 2	Length of the Baſe 666666666
5D	ZD	DC	DN		
15	20	10	1,333333333		
ZD	DC	DC	DN	3	10
20	10	10	5		
ZD	0D	DC	D5		
20	30	10	15		
25	10	10	4	4	12
25	40	10	16		
30	10	10	333333333	5	1333333333
30	50	10	1 666666666		
35	10	10	285714285	6	1428571430
35	60	10	1. 714285715		

This Table conſiſteth of ſix Columns, The firſt four ſhew the Proportion or Ratio to find the Lengths of the Baſes, and the Number in the ſixth Column is the Length of the Baſe for ſo many Spaces as the fifth Column ſignifies.

And by the ſame Reaſon you may find the Lengths of the Baſes for any other Curvilined Trapezia or Hyperbolical Space.

Thus is 666666666 of the ſixth Column (the difference of the two firſt Numbers in the fourth Column) the Length of the Baſe, for the Curvilined compound Trapezia or Hyperbolical Space to repreſent the Logarithm of 2.

And 10 the Length of the Baſe for 3, ſo is 12 for 4: and 1,333333333 for 5, and ſo is 1428571430 for the Length of the Baſe for the Hyperbolical Space for 6. And thus may you do for any other Space or Number.

The Numbers in the fourth Column for 2, 3, 4, 5, 6, *&c.* are in Proportion as $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ *&c.* And added are equal to twice Radius or $20 = D\mathfrak{V}$.

We proceed next to ſhew how to make a Table of Ratios to find the Lengths of both the Perpendiculars (or ſides) of the Compound Curvilined Trapezias.

The

The Fourth Table.

Being a Table of Ratios to find the Length of both the Perpendiculars or Sides of the Compoun Curvilinedd Trapezias or Hyperbolical Spaces.

I		II	III	IV	V
CD— CX	= XD	:DB::	DB:	D5	=XV
10 —333333333	= 666666666	: 10 ::	10 :	15	} 2
CD+ CΠ(=CX)	= DΠ	:DB::	DB:	I Σ	} 2
10 +333333333	=1333333333	: 10 ::	10 :	7, 5	
CD—CN	=ND	:DB::	DB:	DZ	=NE
10 — 5	= 5	: 10 ::	10 :	20	} 3
CD+ CΔ (=CN)	=DΠ	:DB::	DB:	ΔΘ	} 3
10 + 5	=15	: 10 ::	10 :	666666666	
CD—C♋	=♋D	:DB::	DB:	DR	=♋Π
10 — 6	= 4	: 10 ::	10 :	25	} 4
CD+ Cϑ(=D♋)	=Dϑ	:DB::	DB:	ϒϑ	} 4
10 + 6	=16	: 10 ::	10 :	625	
CD—CΩ	=ΩD	:DB::	DB:	D♑	=FΩ
10 —666666666	= 333333333	: 10 ::	10 :	30	} 5
CD+ C♅(=CΩ)	=D♅	:DB::	DB:	♅♎	} 5
10 + 666666666	=1666666666	: 10 ::	10 :	6	

35 } 6
583333333 }

40 } 7
571428571 }

45 } 8
5625 }

50 } 9
555555555 }

55 } 10
55 }

This Table consisteth of five Columns, The first contains the Quantities and Numbers of the first Term in the Proportion; The second Column the Quantities and Numbers of the second Term in the proportion; The third Column the Quantities and Numbers of the third Term in the Proportion; The fourth Column the Quantities and Numbers of the fourth Proportional Number or Term, wherein are Numbers for the Length of both the Perpendiculars for 2, 3, 4, 5, &c. The fifth Column is the Numerical Order of the Compound Curvilined Trapezias or Hyperbolical Spaces of 2, 3, 4, 5, &c.

And

And by the ſame Ratio you may find the Lengths of both the Perpendiculars for any other Compound Curvilined Trapezias to repreſent the Logarithm of any other Number.

By the fourth Column you may perceive the Perpendiculars or Sides (of the Compound Curvilined Trapezias or Hyperbolical Spaces) are in ſuch Proportion the one to the other, as the Number they repreſent are to Unity.

That is to ſay. In the Compound Curvilined Trapezia ΠAVX to repreſent the Logarithm Space of 2. the Perpendi ular XV is to ΠA as 2 is to 1.

And in the Compound Curvilined Trapezia △⊙AVEN to repreſent the Logarithm Space of 3, the Perpendicular NE is in proportion to △⊙ as 3 is to 1.

And ſo the Perpendiculars of the fourth Logarithm Space as 4 to 1. And of the fifth Space as 5 to 1, *&c.* as by the fourth Column of this fourth Table appeareth.

And the Perpendiculars of both the other ſorts of the Curvilined Trapezia or Hyperbolical Spaces are likewiſe in the very ſame Proportion the one to the other, as you may note from what hath been ſaid before of them.

By theſe Tables and by what hath been ſaid formerly; theſe three Curvilined Trapezias have the ſame Properties and Affections as thoſe have in an Hyperbola derived from the Section of a Right Angled Cone.

We ſhall now therefore come to calculate ſome part of this latter Hpperbolical Space before we ſhew, how to do it all at once; that is of the Hperbolical Space △⊙AVEN to calculate the Area of the Space △⊙AC which is equal as before ſhewn to the Space of 5VAB. And when we have ſhewn to calculate this part, we ſhall from this, and what hath been taught how to do the other Part come to derive *Dairy*'s Rule, or rather *James Gregory*'s, which is compriſed in the firſt Inference on his 5*th* Propoſition.

CHAP. VII.

WE have in the 5*th*. Chapter calculated the Area of the Curvilined Trapezia or Hyperbolical Space CAVEN equal to AVEZB for the Logarithm of 2.

In

In this Curvilined Trapezia CAVEN all the Perpendiculars ſtanding upon the Baſe CN are each more than Radius CA (or greater than the Tangent of 45d. 00'.) being ſtill aſcending and affirmative, and therefore by the 1ſt. Table to be continually adas by the Calculation thereof is alſo manifeſt.

We now come to calculate the Curvilined Trapezia C△⊙A part of the Curvilined Trapezia C♈⊙⊙A equal to AVEZB.

In this Curvilined Trapezia C△⊙A (equal to B♑VA) all the Perpendiculars ſtanding upon the Baſe CA are each leſſer than the Radius CA being ſtill deſcending and negative, and therefore to be handled by the firſt Table accordingly.

The Baſe C△ is equal to CN of that Space calculated as before in the 5th. Chapter.

If by the Vertex A you draw a Parallel to the Diagonal CB as Z♈A it is a Tangent to the Curve touching it in the point A, and AB doth cut all the Perpendiculars contrarywiſe to CB. For CX=X♏ is not equal to ♈Π=Π♉ but Π♉ is equal to K♏=KB becauſe CΠ is equal to CX, and the Angle Π♈♉ equal to the Angle KB♏ ſo therefore by the 1 and 4 Table all the Perpendiculars ſtanding upon C♈ are leſſer than Radius. And ſeeing by the ſixth Column of the firſt Table, and alſo by the 4th. of the fourth Table we may find the Length of △⊙. Therefore to know the Area of C△⊙A making C⊕ the firſt Term of an Infinite Series in a continual Proportion, as CA is to C△ that is as 50 to 25 what 25? *facit* 125, as in the Infinite Series of Numbers continually Proportional for the Calculation of the Logarithms of 2 in Chap. 5. you do therefore as there ſaid from 50 (of the ſecond part under the Title, the Quotes to be added) ſubſtract 125, and to that add 4166666666666666, and from that ſubſtract 15625 and ſo on throughout, you ſhall have 4054651081081665 for the Area of the Space C△⊙A equal to BAV5. And thus may you find any other part of C♈ΠA.

We ſhall ſhew how to do it for CΠΞA and CXVA, becauſe from them we ſhall derive *Dairy*'s Rule or rather *James Gregory*'s, for from them we have derived and calculated the Logarithm for 2 to twenty five places, as by the Calculation following next after this will appear.

Now

Now to Calculate the Area of the Curvilined Trapezia or Hyperbolical Spaces CΠΓA and CXVA, you make C or CH the First Term of any Infinite Series, and the Second Term in such a Ratio as ΠC is to CA for your Proportionals of your Infinite Series, and so proceed on as in *Chap.* V. and as here appeareth.

The Infinite Series of Numbers Proportional.				*The Quotes to be added for* CXVA.	
a	333333333333333.	I	A +	333333333333333	+ A
aa.	111111111111111.	II	B —	55555555555555	+ B
aaa	.37037037037037.	III	C +	12345679012345	+ C
a^4	.12345678012345.	IV	D —	.3086419753086	+ D
a^5	..4115226337448.	V	E +	..823045267489	+ E
a^6	..1371742112483.	VI	F —	..228623685414	+ F
a^7	...457247370827.	VII	G +	...65321052927	+ G
a^8	...152[illegible]5790275.	VIII	H —	...19051973784	+ H
a^9	....50805263425.	IX	I +	5645029269	+ I
a^{10}	....16935087808.	X	K —	1693508781	+ K
a^{11}	.....5645029269.	XI	L +	513184479	+ L
a^{12}	.....1881676423.	XII	M —	156806369	+ M

We have but gone through Twelve Steps of this Calculation, to shew the manner thereof; but should you proceed to go through it till it works off, as in *Chap.* V. you may have both the Segment CXVA, and CΠΓA; for if you finish the Calculation, and add up all the Quotes, that Sum will be the Area of CXVA, and be found 405465108108165, as in *Chap.* V. and is equal to the greater Segment WNHE ☽ in the Curvilined Trapezia or Hyperbolical Space CNHE ☽ A, and also WNHE ☽ is equal to C △ ⊙ A equal to AB 5 V.

And if from 333333333333333 you shall substract the Second Number 555555555555555, and to that add the Third Number 2345679012345, and then from that substract the Fourth, and so add and substract according to the Signs + and — throughout, you will have .. 287682072451780 for the Area of the lesser Segment of the Compound Curvilined Trapezia WΠΓA ☽ that is the Area of CΠΣA equal to CWDA equal to V 5 ZE.

 And

And you have not only gotten by this Calculation the Area of each Segment ſeparately, and ſo conſequently the Area of the whole Space, by Addition of thoſe Two, but you have alſo obtained the half of the whole Area at once, for if you ſhall (correſpondent to the Column of the Firſt Table) add the Numbers with + affirmative, they will give you half the Area of the compound Curvilined Trapezia XIIIAV for the Logarithm of 2, which you will ſee preſently exemplified and done to 25 places: And this is the Summ of *James Gregories* Inference on his Fifth Propoſition of his *Exercitationes Geometrica*; and ſo agreeable to the Rule delivered to me as before declared by *Mich. Dairy*: Having acquainted ſeveral Perſons with *Dairy's Rule* in page 1, and ſhewn to them ſome figurative work thereupon in Order to make a Logarithm, I was notwithſtanding ſome time through inadvertency almoſt diſcouraged of ever knowing how to Cube *in infinitum* ſuch a Number as there ſpoken of, neither did any of thoſe to whom I had communicated the ſame, take any ſuch notice thereof (that I know) ſo as to do it. And now I come to ſhew how I overcame that difficulty of Cubing a Range of Figures for 25 Places, which he told me I muſt do *in Infinitum*, before I could make the Logarithm of ſo many places; and to remove this ſtumbling Block (I do confeſs) took up ſome time; for *Dairy* had not then told me a word of ſuch an Authour as *James Gregory*, and I had not known his Works, but for *John Collins*, ſome years after *Dairy's* Death; but before I ever met with *Gregory's* Book, I had obtained my deſire to Cube *in Infinitum* Twenty Five Figures, That is Twenty five 3 by dividing by 9 continually, as in the Calculation following, to find the Logarithm of 2 all at once, which manner diſpatcheth the Calculation much more ſpeedy than the Method of Calculation in the Fifth *Chap.*

And now the Reaſon of Cubing Twenty Five 3, by dividing only by 9 doth follow.

For as much as *Dairy's* Rule before declared, to make the Logarithm of 2, doth bid you to 2, add 1, and from 2 ſubſtract 1; ſo ſhall there be a Reſult or Fraction of $\frac{1}{1}$, and then divide 1 or 100, 000, 000, 000, 0000 by $\frac{1}{1}$, whoſe Quotient is 333333333333333, which Cube *in Infinitum*, it had been as much as if he had ſaid Cube $\frac{1}{1}$ fractionally, which is $\frac{1}{1}$, and divide 10000000000000000 by $\frac{1}{1}$, the Quotient will be

be 37037037037037 for the Cube of a or $\frac{1}{1}$, as in the Operation before going. Now for as much as you would Cube the Number for $\frac{1}{3}$, *viz.* 333333333333333 (which is 1, or 100, 000, 000, 000, 0000, divided by 3) it is, as if you ſhould ſay as 27 to 1000000000000000: what 1? the Quotient will be 37037037037037 for the Cube of a or 333333333333333, as before. Now if you ſhall, as in the Operation before-going, ſet down 333333333333333 (which is equal to $\frac{1}{3}$,) you have no more to do but to divide by 9, for that $\frac{1}{3}$ of $\frac{1}{3}$ is equal to $\frac{1}{9}$, and therefore dividing 333333333333333 by 9, the Quotient will be 37037037037037, as before for the Cube of $\frac{1}{1}$ or a, and ſeeing $\frac{1}{3} \times \frac{1}{3}$ is equal to $\frac{1}{9}$, you have no more to do but to divide continually by 9, and they ſhall all be Proportional Numbers by 7th of the 8th of *Euclid*, and conſequently correſpondent to the odd Powers; for if the Root be multiplied by the Square, that begets the Cube, and the Cube again by the Square, that begets the Fifth Power, and ſo on. So here, for as much as dividing by 9 doth beget the Third Power; if you ſhall therefore continually divide by 9, you ſhall have the reſpective odd Powers accordingly, as is alſo manifeſt by the laſt Figurative Calculation; and all is, for that a 1 doth neither multiply nor divide, and that $\frac{1}{3}$ of $\frac{1}{1}$ is equal to $\frac{1}{9}$, and if you ſhall divide $\frac{1}{9}$ of 1, by 9, the Quotient will be 333333333333333, which is equal to a for the Firſt Number or Root, as before.

Now for as much as to make a Compound Curvilined Trapezia equal to an uncompounded: As for inſtance, to make the Compound Curvilined Trapezia WΠrA ɔ to be equal to the uncompounded CAVEN equal to ABZE = A⊙ϒ for the Logarithm of 2, and to find the Length of the Baſe, and both the Perpendiculars, hath been diſcourſed, and may be ſeen, as in the Third and Fourth Table before-going. We come to handle and calculate the Area of this compounded Curvilined Trapezia WΠrɔA for to make the half Logarithm of 2 at once.

Seeing by the Sixth Column of the Third Table, the Baſe WΠ is 6666666666666666, whoſe half is .333333333333333 for CW or CΠ equal to the Firſt Term in the former Operation (and alſo the ſame as *Dairy*'s Reſult or Fraction of $\frac{1}{7}$) and that I muſt divide in the Ratio of AC to CΠ or CW *in Infinitum*,

finitum, as in the Fifth Chapter, and alſo as in this is ſhewn and taught, for to make the Infinite Series of Numbers Proportional: It will appear that if I do divide 333333333333333 by 9, it will give me the Cube of the Firſt Term, and ſo dividing continually by 9, will produce the Numbers appertaining to the odd Powers, as by the large Calculation to 25 places next following: And ſeeing I am by *Dairy*'s Rule or rather *James Gregory*'s, to divide each of the Numbers of the Infinite Series by the Indices of the odd Powers, it is manifeſt, That this Rule of *Dairy*'s is derivable from the 8 Column of the Firſt Table, for A -|- B -|- C -|- D -|- E -|- F -|- G -|- H -|- I

And A — B -|- C — D -|- E — F -|- G — H -|- I

doth make ^{1}A -|- ^{2}C -|- ^{3}E -|- ^{2}G -|- ^{3}I -|- ^{3}L -|- ^{3}N: And therefore every other Line of the Quotes to be added in the former Operation, doth make half the Logarithm of 2.

In making the Infinite Series in page 34, in Order to make the half Logarithm of 2, to 25 places be very careful to ſet the figures in their due places, and to make that Series you are to divide continually by 9, which being done throughout, you may then prove your work by *Multiplication* in multiplying each line by 9, and if thoſe Multiplications produce the foregoing Numbers you may conclude that part of the work to be well prepared. And ſeeing by the direction over the figurative Work in page 35, you are to divide each of the Numbers in page 34, by 1, 3, 5, 7, 9, &c. You muſt ſo order the Quotes of page 35, that they may lye in the ſame line or range with their reſpective Dividends or Numbers in Page 34: for the better Preventing miſtakes, the letter Figures do repreſent the Diviſors proper to each line; and would you make the Logarithm of 2, according to that Method in page 34 and 35, for 7 or 8, places only, you may very well produce it in half an hours time as by that Calculation is very perceptible. And ſome that have had thoſe two ſheets I formerly Printed as a Specimen hereof, have told me they have done the ſame, and were very ſolicitous I would as ſoon as I could, publiſh the remainder, which at length as time and leiſure hath permitted is done: and though I have not here inſerted many Examples; yet by what are herein done you may perceive how to proceed for any other Number propoſed. And with the direction and reference in page 46, thoſe that are willing and curious herein may make a Logarithm for

any

any natural Number desired. I have not added hereto any Table of Logarithms at this time, and what I may do hereafter in order thereunto I do not presume to promise. I doubt not but some may both examime some Table or other, or make by this Method one *De Novo*, and satisfie themselves about the same, and some have told me since my communicating this Method unto them, that if the first makers of Tables of these Numbers had made them by such easie ways, they did not doubt but their Tables might have been somewhat more exact. Howsoever it pleased God who is the giver of every good and perfect Gift, to raise and endue such men with great ability and patience to perform those Tables with so much difficulty and labor as their Methods did require, and for common Uses sufficient. And with such Eagerness did that Age embrace and pursue the Invention of these Numbers that *Ullach* a Dutchman had exhibited a Table of Logarithms to 10 places for 100000 before the Learned *Henry Brigg*'s Table, which he had in part done to 15 places, could be accomplished by him. So exceeding glad were they of the Invention. And the Learned *Henry Briggs* in his Epistle Dedicatory to Our Most Gracious King's Father when Prince of *Wales*, saith, that amongst the Antients there is not found any Footsteps of these Numbers; of whose Construction and Uses the said *Henry Briggs* hath written in his *Arithmetica Logarithmica* most learnedly and copiously, and now follows the figurative part of making the Logarithm of 2 to 25 Places.

The

The Infinite Series or Numbers continually Proportional. These Numbers are continually divided by 9, in order to make the Half Logarithm of 2.

a	333333333333333333333333	I
a	370370370370370370370370	III
a^5	411522633744855967078 19	V
a^7	457247370827617741 1980	VII
a^9	508052634252908601331	IX
a^{11}	564502926947676223 70	XI
a^{13}	627225474386306915 2	XIII
a^{15}	696917193762563239	XV
a^{17}	774352437513959 15	XVII
a^{19}	860391 5972377324	XIX
a^{21}	955990663597480	XXI
	1062211848441 64	XXIII
	1180235387 1574	XXV
	1311372652397	XXVII
	145708072489	XXIX
	16189785832	XXXI
	1798865092	XXXIII
	199873899	XXXV
	22208211	XXXVII
	2467579	XXXIX
	274175	XLI
	30464	XLIII
	3385	XLV
	376	XLVII
	42	XLIX

Differentia 1
Unitas 1
Numerus Propositi. 2
Summa 3

$a = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ aaa

You may perceive that if 1 be added to 2, and subſtracted from it, it leaveth a Reſult of $\frac{1}{3}$, which multiplied into it ſelf, maketh $\frac{1}{9}$, and therefore theſe Numbers are continually divided by 9.

Theſe

These Numbers are Quotes from those on the opposite Side, by dividing them by 1, 3, 5, 7, 9, &c. and are $^{1}A+$ $^{3}C+$ $^{5}E+$ $^{7}G+$ $^{9}I+$ $^{11}L+$ $^{13}N+$, &c. Correspondent to the last preceding Calculation, which added make half the Area of the Compound Curvilined Trapezia XΠΓAV for the Half Logarithm of 2 to 25 Places.

3333333333333333333333333	A
1234567901234567901234567	C
823045267489711934156 4	E
65321052975373963028 3	G
5645029269476762237 0	I
513184479043342021 6	L
48248113414331301 2	N
4646114625083754 9	P
455501433831740 7	R
45283768275670 2	T
4552336493321 3	W
461831238452 9	Y
47209415486 3	
4856935749 6	
502441629 3	
52225115 6	
5451106 3	
571068 3	
60022 2	
6327 1	
668 7	
70 8	
7 5	
8	

Half the Logarithm of 2. {346573590279972654708616 0

The Logarith. of 2. {693147180559945309417232 1

Thus

Thus have we Calculated the Logarithm for 2 to 25 Places, after *Dairy*'s Rule, or rather *James Gregory*'s, which Method maketh far greater diſpatch than that in *Chap.* V. for this Calculation though to 25 Places, is ſooner performed than that of 15 Places in *Chap.* V. as by comparing them is very perſpicuous and manifeſt.

And now we have exemplified the Rule *Dairy* declar'd, and I am apt to believe he had ſtudied well *Gregory*'s ſaid *Exercitationes*, though he was not pleaſed to tell any more thereof, but that others ſhould take pains therein as well as he, and truly if *John Collins* had not acquainted me with *Gregory*'s Works, I had done the Work, but not with that ſatisfaction I met with from *James Gregory*'s Books; and here you have it in a more familiar Diſcourſe and Dialect than that of *James Gregory*'s, being altogether Analitical, and if any Letter or Symbol be miſtaken in his, it is very great Study and Labour to find, and to ſet it to rights.

I find *James Gregory* hath calculated the Hyperbolical Logarithm for 2 in his *Vera Circuli & Hyperbola Quadratura* to 25 Places, which agreeth with this Calculation, but to 17 Places I have not raiſed the Logarithm for 2 to his Doctrine in that Book, but am ſatisfied this Calculation for the Logarithm of 2 in this Chapter is true to an Unite in the 25th Place, and may be in Two hours very well Examined by any one that will take the pains to do it, and they ſhall find it to be as herein Calculated. And to this I have the Concurrence of the moſt ingenuous and laborious Mr. *Abraham Sharp*, who (from the Occaſion of my publiſhing formerly two ſheets of the Praxis hereof as a Specimen) hath ſhewn me his Calculation of the Logarithm of 2, and ſome others to forty Places, the like I ſuppoſe not hitherto heard of or ſeen. Without all doubt *Gregory* found that *Mercators* Lucky Invention of Squaring the Hyperbola, was of far more diſpatch than that of his *Vera Circuli & Hyperbola Quadratura*, or elſe he would not have Writ upon *Mercator*: But ſo excellently hath *Gregory* Illuſtrated *Mercator*, that a better way of Squaring the *Hyperbola* I ſuppoſe hath not nor may be found.

We shall for the future keep to this last Method for Calculating those Examples hereafter following. And now we proceed to calculate a Logarithm for 1 $\frac{1}{4}$, for having got the Logarithm of 2, we have the Logarithm of 4, and all the Powers thereof; so then if we make a Logarithmof 1 $\frac{1}{4}$, and add to it the Logarithm of 4, we shall have the Logarithm of 5, because that 4 X 1 $\frac{1}{4}$ makes 5; that is, 4 multiplyed by 1 $\frac{1}{4}$, makes 5: And this Method I took to make the Logarithm of 5, and having got the Logarithm of 5, to the Logarithm of 5 add the Logarithm of 2, and you have the Logarithm of 10. And when I have shewn this, I shall produce *Brigg's* Logarithm of 2, by one single Division; for that all sorts of Logarithms are Proportional the one to the other; we therefore now hasten to make the Logarithm for 1 $\frac{1}{4}$.

e Calculation of the Logarithm for 1 $\frac{1}{4}$ o 25 Places.

Infinite Series or Numbers continually roportional; these Numbers are continuly divided by 81.

1111111111111111111111111	I
13717421124828532235940	III
1693508780843028671111	V
209075158128768971 8	VII
25811747917131972	IX
3186635545324 93	XI
393411795719 1	XIII
48569357496	XV
5996216 97	XVII
7402737	XIX
91392	XXI
1128	XXIII
14	

These Numbers are Quotes from those on the opposite Side, they being divided by 1, 3, 5, 7, 9, &c. *which added, make half the Logarithm for* 1 $\frac{1}{4}$ *to* 25 *Places.*

1111111111111111111111111	A
4572473708276177411980	C
338701756168605734 22	E
29867879732681281 7	G
2867971990792441	I
289694140484 08	L
30262445824 5	N
3237957166	P
35271865	R
389618	T
4352	W
49	Y
1	

1115717755571048778831475
This Numb. is $\frac{1}{4}$ the Log. for 1 $\frac{1}{4}$
2231435513142097557662951
This Number is the Log. of 1 $\frac{1}{4}$

Adding 1 to 1 1/3, it maketh 2 1/3, and subſtracted from it, leaveth 1/3, which Reſult maketh a Fraction of 1/3, for 2 1/3 being reduced into Fourths, make 9/4, ſo the Reſult of 1/3 and 9/4 (rejecting the Denominators) is 1/3 as above, which Squared maketh 1/9; ſo are theſe Numbers therefore continually divided by 81, to make the Infinite Series. By Decimals it preſently ſheweth it ſelf to be a Fraction of 1/9. Thus the Difference or Numerator is 25 the Summ or Denominator 2,25, Which Decimal Fraction 2,11 is equal to 1/9.

Differentia	25
Unitas	1
Numerus Propoſitus	1,25
Summa	2,25

To divide the Numbers on the other ſide (to make the Infinite Series) by 81, is eaſie enough, for it is but dividing twice by 9, or taking one Ninth part twice, and rejecting or cancelling the firſt, ſo is it very readily done, and the whole Operation hereof may very well be performed in two hours time; and thus have we got the Logarithm for 1 1/3 to 25 places, and now ſhall proceed to make the Logarithm of 10, which is by adding together the Logarithm of 2, 3 times, and that makes the Logarithm of 8, and that added to the Log. of 1 1/3, makes the Log. of 10; and the Logarithm of 2 ſubſtracted from the Logarithm of 10, leaveth the Logarithm of 5, and is to the ſame effect as is before.

Logarithm	of	2.	6931471805599453094172321
Logarithm	of	8.	2079441541679835928516963
Logarithm	of	1 1/3.	2231435513142097557662951
Logarithm	of	10.	2302585092994045684017991 4
Logarithm	of	5.	1609437912434100374600 7593

We have now made and exhibited the Logarithms of 2, 5, and 10, and from theſe you may make all their Compoſites.

And now we proceed to make the Logarithm of 3 to 25 Places, which we ſhall ſhew two ways, firſt, all at once from a Compound Curvilined Trapezia or Hyperbolical Space; ſecondly, by a Compoſition of 2 Logarithms, *viz.* of 2 and 1 1/3, for that 2 X 1 1/3 maketh 3, and this latter we chiefly recommend. The Compound Curvilined Trapezia or Hyperbolical-Space N△⊙AVE we have in the foregoing *Chapter* ſhewn to be equal to the uncompounded CAVFO, and alſo equal to AVF⊙B,

we

we ſhall Calculate the Logarithm for 3, according to the Compounded Space, and by the Third and Fourth Tables, you may know the Lengths of the Baſe and Perpendiculars.

The Baſe N△ is 10, therefore CA = ⅓ = CX. Now for as much as *Dairy*'s Rule of adding to, and ſubſtracting 1 from 3, produces the half Length of the Baſe agreeable to the Third Table, we ſhall ſhew how to Calculate the half of the Logarithm for 3, as we did for the half Logarithm of 2.

Adding 1 to 3 it makes 4, and ſubſtracting 1 from 3, leaveth 2, which maketh a Reſult or Fraction of $\frac{2}{4} = \frac{1}{2}$.

Now dividing 1 and 25 Cyphers by $\frac{1}{2}$, the Quotient is 5, for the firſt Term of your Infinite Series. Now for as much as $\frac{1}{2} \times \frac{1}{2}$ maketh $\frac{1}{4}$, the ſecond Term muſt be therefore $\frac{1}{4}$ of the Firſt, and ſo on, as was diſcours'd before in making the half Logarithm of 2.

Having made the Infinite Series as followeth, you divide each of thoſe Numbers (which as before Taught are proportional) by 1, 3, 5, 7, 9, &c. and theſe added, make half the Area of N△♦AVE for the half Logarithm of 3.

The Infinite Series or Numbers continually Proportional. These Numbers are continually divided by 4, in order to make the ½ Log. of 3 to 25 Places.

a	500000000000000000000000
aaa	125
$aaaaa$	3125
a^{7}	78125
a^{9}	1953125
a^{11}	48828125
a^{13}	1220703125
a^{15}	30517578125
a^{17}	762939453125
a^{19}	19073486328125
a^{21}	476837178203125
	11920928955078125
	298023223876953125
	74505805969238281
	18626451492309570
	4656612873077392
	1164153218269343
	291038304567336
	72759576141834
	18189894035458
	4547473508864
	1136868377216
	284217094304
	71054273576
	17763568394
	4440892099
	1110223025
	277555756
	69333936
	17333485
	4333371
	1083343
	270836
	67709
	16927
	4232
	1058
	264

Differentia 2
—
Unitas — 1
Numerus Propositus 3
—
Summa — 4

$\overset{a}{=\tfrac{1}{2}} \times \overset{aaa}{\tfrac{1}{2}} = \tfrac{1}{4}$

These

ese Numbers are Quotes from those on the opposite side, those being divided by 1, 3, 5, 7, 9, &c. which added make half the Log. of 3 to 25 Places.

I	50000000000000000000000
III	41666666666666666666666
V	625
VII	11160714295714285714285
IX	21701388888888888888888
XI	44389204545454545454545
XIII	93900240384615384615
XV	20345052083333333333
XVII	44878791360294117765
XIX	10038677014802631 58
XXI	22706531343005952 3
XXIII	5183012589164402 2
XXV	1192092885507812 5
XXVII	275947429515697 5
XXIX	64229143075929 5
XXXI	15021331848636 7
XXXIII	3224706722028 3
XXXV	831538913049 5
XXXVII	19664750308 61
XXXIX	4664075392 94
XLI	1109139880 21
XLIII	264387994 70
XLV	63159354 29
XLVII	15117930 55
XLIX	3625218 04
LI	870763 16
LIII	209476 04
LV	50464 69
LVII	12163 85
LIX	2937 88
LXI	710 39
LXIII	171 96
LXV	41 67
LXVII	10 10
LXIX	2 45
LXXI	59
LXXIII	15
LXXV	3
Half the Log. of 3.	54930614433405484569762 26
The Logarithm of 3.	10986122886810969139524 52

We

We shall now proceed to make the Logarithm for 3 the second way, which is from the Logarithms of 2 and $\frac{1}{2}$.

The Infinite Series or Numbers continually Proportional. These Numbers are continually multiplied by 4, in order to make the Half of the Logarithm of $1\frac{1}{2}$ *to 25 Places.*

a	2000000000000000000000000
aaa	8
aaaaa	32
a^{7}	128
a^{9}	512
a^{11}	2048
	8192
Differentia $\frac{1}{4}$	32768
	131072
Unitas — 1	524288
Numerus Propositus $1\frac{1}{2}$	2097152
	8388608
Summa $2\frac{1}{4}$	33554432
	134217728
	536870912
	2147483648
	8589934592

$= \frac{a}{1} \times \frac{aaa}{1}$

To make this Infinite Series, I should divide by 25 continually, but if you multiply by 4, and transfer it answerable, it will be the same thing: Because $\frac{1}{4}$ of 10 is $\frac{2}{100}$, and that multiplied in it self; is $\frac{4}{10000}$. Therefore multiplying by 4 and dividing by 100, is the same thing as multiplying by 25; And thus this Infinite Series is made very speedily, in order to make the half of the Logarithm for 1 and $\frac{1}{4}$.

These

These Numbers are Quotes from those on the opposite side, they being divided by 1, 3, 5, 7, 9, 11, &c. *and added, make half the Logarithm for* 1 $\frac{1}{10}$ *to* 25 *Places.*

I	2000000000000000000000000
III	2666666666666666666666666
V	64
VII	1828571428571428571 4
IX	568888888888888888
XI	186181818181818181 8
XIII	63015384615384 6
XV	21845333333334
XVII	7710117647 06
XIX	2759410520 4
XXI	99864380 9
XXIII	3647220 9
XXV	134217 7
XXVII	4971 1
XXIX	185 1
XXXI	6 9
XXXIII	3

Half the Logar. of 1 $\frac{1}{10}$.	202732554054082190989006 5
The Logarithm of 1 $\frac{1}{10}$.	405465108108164381978013 1
The Logarithm for 2.	693147180559945309417232 1
The Logarithm for 3.	1098612288668109691395245 2

Having now made the Logarithm for 1 $\frac{1}{10}$, you add to it the Logarithm of 2, and that makes the Logarithm for 3, which will be found as before to be the same Number.

And now we proceed to make the Logarithm for 7, and then we shall have all to 11. In order thereunto, we make the Logarithm for 1, $\frac{4}{5}$ or 1,, $\frac{8}{10}$, and add that to the Logarithm of 5, and it will produce the Logarithm of 7, for that 1 $\frac{4}{5}$ multiply by 5, maketh 7, or 1 $\frac{4}{5}$ X 5 = 7.

The

The Infinite Series or Numbers continually Proportional: These Numbers are continually divided by 36, in order to make half the Logarithm of $1\frac{2}{6}$ *or* $1\frac{4}{12}$.

a	1666666666666666666666666
.	2777777777777777777777777
aaa	46296296296296296296296
.	77160493827160493827160
aaaaa	1286008230452674897119
:	2143347050754458161860
a ^{7}	35722450845907636031
:	59537418076512726720
a ^{9}	992290301275212112
:	16538171687920201 9
a	275636194798670 03
	4593936579977864
	765656096662972
	127609349443829
	21268224907304
	3544704151217
	590784025203
	98464004200
	16410667366
	2735111228
	455851871
	75975312
	12662552
	2110425
	351737
	58623
	9737
	1623
	270

Differentia 4
Unitas 1
Numerus Propositus 1 4
Summa — 2 4

$= \frac{a}{6} \times \frac{1}{6} = \frac{aaa}{1\frac{4}{12}}$

This Series is made by dividing Twice by 6, which is all one as if you divided at once by 36, and so every other Number is the proper Number of the Series to be divided by 1, 3, 5, 7, 9, 11, &c. as in the other side to make the half Logarithm for $1\frac{2}{6}$.

☞ The Logarithm of 5 in page 38 being put in the room of that in page 45 will produce 1945910149055313305105353 for the true Logarithm of 7. Those two last Numbers in page 45 being part mistaken. *These*

These Numbers are Quotes from those on the opposite side, those being divided by 1. 3. 5. 7. 9. 11, &c. And added make half the Logarithm of $1\frac{2}{1}$ or $1\frac{4}{1}$.

I	166666666666666666666666666
III	154320987654320987654320
V	25720164609053497942 4
VII	5103207263701090863
IX	1102544778194680 14
XI	250578358907881 9
XIII	58896622820229
XV	1417881660487
XVII	34752001483
XIX	863719335
XXI	21707237
XXIII	550545
XXV	14069
XXVII	360
XXIX	9
Half the Logarithm of $1\frac{4}{1}$.	16823611831060646525229 72
The Logarithm of $1\frac{2}{1}$ or $1\frac{4}{1}$.	33647223662121293050459 44
The Logarithm of 5.	16094379124341003168895254
The Logarithm of 7.	19459101490553132473941198

Having by this Calculation made the half Logarithm of $1\frac{4}{5}$. if we double it, and to that add the Logarithm of 5 that Addition will produce the Logarithm of 7 as was required. And now we have all the Logarithms to 11, and to make the Logarithms from 10 to 100, it will not be much difficult to proceed after the foregoing Methods, as to make the Logarithm of 11, you have for the first Term *a*, the result or fraction $\frac{5}{6}$, and for *a a a*, it will be $\frac{25}{26}$, which is very easie to work. And for the Logarithm of 13, you make it of 12 multiplyed by $1\frac{1}{12}$. And so it is for the first Term *a*, the result or fraction $\frac{3}{25}$, and for the second *a a a*, it is $\frac{9}{625}$, which 625 is = to $\frac{5}{8}$, and so may you make many easie compendiums for the Prime Numbers between 10 and 100, and also not with great difficulty from 1000 to 10000, and when you have made some Logarithms you will perceive how the differences arise, and having for Composites, Logarithms in a readiness, greater and lesser than the Prime or Incomposite very near, it will be by the Difference no great difficulty to make a Logarithm for such a Prime very readily and easily. And they that are curious herein may have Compendiums hereof in *James Gregory*'s aforesaid *vera Circuli & Hyperbolæ Quadratura* to make Logarithms for Prime or Incomposite Numbers, to which I shall refer him; and here I shall content my self to have exemplified *James Gregory*'s Method in his *Exercitationes Geometricæ* to so many Examples of Logarithms as I have herein Calculated to 25 Places, and shall in the next Chapter shew how to produce from these Geometrical Logarithms *Brigg*'s Logarithms.

CHAP.

CHAP. VIII.

HAving in the Preceeding Chapter made the Logarithms for 2, 3, 4, 5, 6, 7, 8, 9, and 10, according to the Geometrical Figure or Hyperbola, I require the Logarithm of 2 according to *Brigg*'s Table. For as much as all Logarithms are Proportional, it is as the Hyperbolical Logarithm of 10, is to its Logarithm of 2 :: So is *Brigg*'s Logarithm of 10 to his Logarithm of 2. The Operation followeth,

This Divisor is half the Logarithm of 10, according to the Hyperbola.

Diviſor

. . . . . .

1 1 5 1 2 9 2 5 4 6 4 9 7 0 2 2 8 1

Quotient.

3 0 1 0 2 9 9 9 5 6 6 3 9 8 1 1 9 0

This Quotient is the Logarithm of 2, according to *Briggs* his Table.

By this Diviſion it doth appear, that this Quotient doth agree with *Briggs* his Table of Logarithms for his Logarithm Number of 2, whereby it is apparent he did produce the Logarithm for 2 to 15 places very true, though I have been told it was eight Perſons work for a years time after his Method which was by large and many Extractions of the Square Root, and if it was ſo to 15 places, it would have been very tedious (if not impoſſible) for them to have produced the Logarithm of 2 to 25 places, as before herein is ſhewn and done by us, and both the Hyperbolical and *Briggs*'s Logarithm to 25 places may very well be calculated and done according to the foregoing Method in half a days time, by which Method herein before going one may make a Table of Logarithms in a ſhort Space to what *Pardie* in his Elements of Geometry (a French Author hath declared, for he ſaith, he knew more than 20 perſons engaged for 20 years with indefatigable aſſiduity to calculate the Logarithms. He doth not ſay to how many places: But the greateſt Radius that I have ſeen of any French Author is but 11 places, which I ſuppoſe muſt be but the ſame as *Vulach*'s. And the Logarithm for 2, 3, 4, or 5, *&c.* to 11 places according to the Method in this Book may be very well done and performed in leſs than two hours time.

This Dividend is compounded of half the Hyperbolical Logarithm of 2, and *Briggs* his Logarithm of 10.

Divide

Dividend

. . . . . .

3465735902799726 4, 0000000000000000

1185820330865797

3453378436877419

115079334388337338

114630052036052851

110137228513207981

652089932847592810

7644365959908 1405

7356106809260 0364

4583515302778 9954

112963766328792697

9347437144060543 1

137096772084362 62

219675174346603 39

104445919096901109

829590512169056 1

The Reader may now see that Logarithms derived from this Figure or the Hyperbola are not only more perceptible and intelligible, but with far more Certainty and Expedition produced than what was known in former times.

The Divisor in the foregoing work differs 2 Unites in the 18place, from the half Logarithm of 10 before herein calculated, and the Reason is, that I took *Gregory*'s Logarithm of 10 in his *Vera Circuli & Hyperbolæ Quadratura de bene esse*, and having calculated the half Logarithm of 2 as before, I was very desirous to see if we could produce *Brigg*'s Logarithm of 2 to 15 places, as by the Division is manifest, and this I did, long before I met with *Gregory*'s other Book of his *Exercitationes Geometricæ*; for since I got that Book I did calculate *De Novo* the Logarithm of 10 to 25 places according to his Doctrin in that Book, and as before herein is done. And the Calculation of the Logarithm of 10 as before doth agree with *Gregory*'s former Book but to 17 places; howsoever the Division before going is sufficient to produce *Brigg*'s Logarithm for 2 to 15 places, and if any shall be so curious to produce *Brigg*'s Logarithm for 25 places,

places, he may rely on the foregoing Examples herein, and may in 4 hours time examine the foregoing Calculations thereof, and in as little time produce *Brigg*'s Logarithm for them to the like Number of places.

Having this Division ready done, long before the publishing hereof, I have contented my self to insert it here, whereby the studious may soon perceive what to do further to gratify himself herein.

I do not add hereto any Table of Logarithms, that being not my design at this time, but only to shew how *Brigg*'s or any other Logarithms may be derived from the Doctrin before going, and also for the curious at his will and pleasure to examine whether any Logarithms formerly published be truly made or not.

As for the various Uses of Logarithms I add none here, but refer the Reader to such Authors (whereof there is plenty,) who have long before written largely and learnedly, as the first Inventor the famous Lord *Neper*, *Henry Briggs*, *Edm. Gunter*, *Rich. Norwood*, *Wingate*, and divers others; as also my Father *John Speidell*, in which the Reader may meet with many excellent Uses of the Logarithms in all parts of the Mathematicks; and I do find my Father printed several sorts of Logarithms, but at last concluded that the Decimal or *Brigg*'s Logarithms were the best sort for a standard Logarithm; and did also print the same several ways, so ordered, whereby they might be applyed to Arithmetical Questions and other Operations for the Solution thereof with ease and readiness.

F I N I S.

E R R A T A.

Page 9. Line 5, read 7 *and* 8 *of Euclid.*
Page 31. Line 25. for $\frac{1}{7}$. read $\frac{3}{7}$.
Page 38 Line 10. for 2 25. read $\frac{1}{25}$.
Page 39. Line 5. for $2\frac{1}{4}$ $1\frac{1}{2}$. read $2\frac{1}{4}$ $1\frac{3}{4}$.

To the READER.

HAving for some Years past shewn to several Persons the Praxis of the following Treatise, and also communicated to them somewhat of the Doctrin leading thereunto, I was often desired not to let them sleep in oblivion, but to publish the same, which was first promoted by my honoured Friend Mr. Peter Hoot Merchant, and seconded by my loving Friend Mr. Reeve Williams, or else they had not seen the Publique; what I have done therein I desire thee to take in good part, being also at proportional charges my self, besides my composing thereof to make it communicable to thee, rather than such an easie and certain way to make Logarithm Numbers (to so many Places) should not be known in our Native Tongue. I have called them Geometrical Logarithms, for that the first Inventors of those Numbers had not adapted Geometrical Figures to them. But the Scheme hereunto annexed having such Properties and Affections as Logarithm Numbers have, hath made me so style them. What I have done herein is to gratifie such who have a Curiosity to examine Logarithm Tables, and to make Logarithm Number to so small Radiuses as are so often printed for common uses with brevity and exactness. Two sheets of the Praxis hereof were printed some time before the rest, which having found kind acceptance with divers, induced me also to let the Remainder be published; and before the printing thereof one was writing upon those two sheets, and was so fair to desire my consent to publish it, which I readily gave, for that I knew him able enough to do it, and when to be at Leisure my self to attend the publishing of the Residue, I knew not. But that me being performed by him, I desire thee to accept of what is done herein as time and leisure hath permitted. I shall not need to write how needful Logarithm Numbers are in those great and useful Arts of Navigation, Astronomy, Dyalling, Fortification and Gunnery, Surveying, Guaging, Interest and Annuities, &c. When, as there are so many Books written and published thereof, not only in our own Language but in many others. And truly the first Inventors thereof are not a little to be had in reverence for making and perfecting those Numbers with so much Labour, as those Methods by which they derived them did require. Here thou mayst make a Logarithm to

7 or 8

7 or 8 places readily and easily; but to 25 places would have been very difficult, if not impossible, for the first Inventors to have produced after their ways. If any thing herein shall offer, whereby thou mayest make farther Improvement, Let the Publique share of the benefit thereof. Thus wishing thee good Success in all thy Studies, is most earnestly desired by,

H

London,
March, 26. 1688.

E. Speidell.

Advertisement of Characters, *or* Symbols, *used in this Treatise.*

+	signifieth	More, or Addition,
×	signifieth	Multiplication,
−	signifieth	Less, or Substraction,
=	signifieth	Equal.

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Geome-

The Infinite Series of Numbers Proportional.			*The Quotes to be Added.*	
A	50,0000000000000	I	*50,0000000000000	A
B	25	II	—125	B
C	125	III	* 41666666666666	C
D	625	IV	—15625	D
E	3125	V	* 625	E
F	15625	VI	— 2604166666666	F
G	78125	VII	* 1116071428571	G
H	390625	VIII	— 48828125	H
I	1953125	IX	* 217013888888	I
K	9765625	X	— 9765625	K
L	48828125	XI	* 44389204545	L
M	244140625	XII	— 20345052083	M
N	1220703125	XIII	* 9390014037	N
O	6103515625	XIV	— 4359654017	O
P	30517578125	XV	* 2034505206	P
Q	15258789062	XVI	— 953674316	Q
R	7629394531	XVII	* 448787914	R
S	3814697265	XVIII	— 211927625	S
T	1907348632	XIX	* 100386770	+
V	953674316	XX	— 47683715	
W	476837158	XXI	* 22706531	
X	238418579	XXII	— 10837208	
Y	119209289	XXIII	* 5183013	
Z	59604644	XXIV	— 2483527	
	29802322	XXV	* 1192093	
	14901161	XXVI	— 573122	
	7450580	XXVII	* 283355	
	3725290	XXVIII	— 133046	
	1862645	XXIX	* 64229	
	931322	XXX	— 31044	
	465661	XXXI	* 15021	
	232831	XXXII	— 7276	
	116415	XXXIII	* 3527	
	58207	XXXIV	— 1712	
	29103	XXXV	* 831	
	14551	XXXVI	— 404	
	7275	XXXVII	* 196	
	3637	XXXVIII	— 96	
	1818	XXXIX	* 46	
	909	XL	— 22	
	454	XLI	* 11	
	277	XLII	— 5	
			* 2	
			— 1	

10, 00000000 =
The whole Summ
A—B: A :: A =

6, 66666666 =
A—C: A :: A =
Impares

3, 333333333 =
Pares
B—D: B :: B: =

A—B. C—D. E—F. G—H
= 3, 333333333

A—B+C—D+E—F+
= 3, 333333333

A+B+C+D+E+F+G
A—B+C—D+E—F+G
= A+ [illegible]

Logarithm of 2.	693147180 59945
Half the Logarithm of 3	549306144334055
Logarithm of 3	1098612288668110
Logarithm of the differ. between 2 & 3 or the Log. of 1, 5.	405465108108165
	143841036225890
	287682072451780